

DECAY SPECTRUM OF  $K^+ \rightarrow e^+ \nu_e \gamma$ C.H. CHEN<sup>1</sup>, C.Q. GENG<sup>2</sup> and C.C. LIH<sup>3</sup><sup>1</sup>*Department of Physics, National Cheng-Kung University, Tainan 701, Taiwan*<sup>2</sup>*Department of Physics, National Tsing-Hua University, Hsinchu 300, Taiwan*<sup>3</sup>*General Education Center, Tzu-Chi College of Technology, Hualien 970, Taiwan*

The form factors of the  $K^+ \rightarrow \gamma$  transition are studied in the light-front quark model and chiral perturbation theory of  $O(p^6)$ . The decay spectrum of  $K^+ \rightarrow e^+ \nu_e \gamma$ , dominated by the structure dependent contribution, is illustrated in both models.

It is known that the decay of  $K^+ \rightarrow e^+ \nu_e \gamma$  receives two types of contributions: “inner bremsstrahlung” (IB) and “structure-dependent” (SD) <sup>1,2</sup>. The former is helicity suppressed and contains the electromagnetic coupling constant  $\alpha$ , while the latter gives the dominant contribution to the decay rate as it is free of the helicity suppression. In the standard model (SM), the decay amplitude of the SD part involves vector and axial-vector hadronic currents, which can be parametrized in terms of the vector form factor  $F_V$  and axial-vector form factor  $F_A$ , respectively. However, the experimental determinations on these form factors are poorly given and model-dependent <sup>3,4,5</sup>. In particular, the experimental results on the decay rate of  $K^+ \rightarrow e^+ \nu_e \gamma$  in Ref. <sup>3,4,5</sup> were based on the assumption of  $F_V$  and  $F_A$  being some constant values in the chiral perturbation theory (ChPT) at  $O(p^4)$  <sup>6</sup>. In the ongoing data analysis of the E949 experiment at BNL, more precision measurements on the decay of  $K^+ \rightarrow e^+ \nu_e \gamma$  are expected <sup>7</sup> and thus, the model-independent extractions of the SD form factors are possible. Theoretical calculations of  $F_V$  and  $F_A$  in the  $K^+ \rightarrow \gamma$  transition have been previously done in the ChPT at  $O(p^4)$  <sup>6</sup> and  $O(p^6)$  <sup>8,9</sup> as well as the light-front quark model (LFQM) <sup>10</sup>. However, the results have not been fully applied to the decay of  $K^+ \rightarrow e^+ \nu_e \gamma$  yet.

In this talk, we will present our recent results <sup>11</sup> on the transition form factors of  $K^+ \rightarrow \gamma$  in the ChPT of  $O(p^6)$  and light-front quark model (LFQM). We will show the spectrum of the differential decay branching ratio of  $K^+ \rightarrow e^+ \nu_e \gamma$  as a function of  $x = 2E_\gamma/m_K$ .

We start with the amplitude of the decay  $K^+ \rightarrow e^+ \nu_e \gamma$  in the SM, given by <sup>2,6,12</sup>

$$M = M_{IB} + M_{SD},$$

$$M_{IB} = ie \frac{G_F}{\sqrt{2}} \sin \theta_c F_K m_e \epsilon_\alpha^* K^\alpha, \quad M_{SD} = -ie \frac{G_F}{\sqrt{2}} \sin \theta_c \epsilon_\mu^* L_\nu H^{\mu\nu}, \quad (1)$$

where  $K^\alpha = \bar{u}(p_\nu)(1 + \gamma_5) \left( \frac{p_K^\alpha}{p_K \cdot q} - \frac{2p_e^\alpha + q\gamma^\alpha}{2p_e \cdot q} \right) v(p_e)$ ,  $L_\nu = \bar{u}(p_\nu) \gamma_\nu (1 - \gamma_5) v(p_e)$ ,

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$H^{\mu\nu} = \frac{F_A}{m_K} (-g^{\mu\nu} p_K \cdot q + p_K^\mu q^\nu) + i \frac{F_V}{m_K} \epsilon^{\mu\nu\alpha\beta} q_\alpha p_{K\beta} \epsilon_\alpha$  is the photon polarization vector,  $p_K$ ,  $p_\nu$ ,  $p_e$ , and  $q$  are the four-momenta of  $K^+$ ,  $\nu_e$ ,  $e^+$ , and  $\gamma$ , and  $F_K$  and  $F_{A(V)}$  are the  $K$  meson decay constant and the axial-vector (vector) form factor corresponding to the axial-vector (vector) part of the weak currents, respectively, defined by

$$\begin{aligned} \langle 0 | \bar{s} \gamma^\mu \gamma_5 u | K^+(p_K) \rangle &= -i F_K p_K^\mu, \quad \langle \gamma(q) | \bar{u} \gamma^\mu s | K(p_K) \rangle = i e \frac{F_V}{m_K} \epsilon^{\mu\alpha\beta\nu} \epsilon_{*\alpha} q_\beta p_\nu, \\ \langle \gamma(q) | \bar{u} \gamma^\mu \gamma_5 s | K(p_K) \rangle &= e \frac{F_A}{m_K} [(p \cdot q) \epsilon^{*\mu} - (\epsilon^* \cdot p) q^\mu], \end{aligned} \quad (2)$$

with  $p = p_K - q$  being the transfer momentum. We note that  $\mathcal{M}_{IB}$  in Eq. (1) is suppressed due to the small electron mass  $m_e$ . In the decay of  $K^+ \rightarrow e^+ \nu_e \gamma$ , the form factors  $F_{A,V}$  in Eq. (2) are the analytic functions of  $p^2 = (p_K - q)^2$  in the physical allowed region of  $m_e^2 \leq p^2 \leq m_K^2$ . The relation between the transfer momentum  $p^2$  and  $x$  is given by  $p^2 = m_K^2(1 - x)$ .

At  $O(p^6)$  in the ChPT, one obtains that <sup>11</sup>

$$\begin{aligned} F_V(p^2) &= \frac{m_K}{4\sqrt{2}\pi^2 F_K} \left\{ 1 - \frac{256}{3} \pi^2 m_K^2 C_7^r + 256 \pi^2 (m_K^2 - m_\pi^2) C_{11}^r + \frac{64}{3} \pi^2 p^2 C_{22}^r \right. \\ &\quad - \frac{1}{16\pi^2 (\sqrt{2} F_K)^2} \left[ \frac{3}{2} m_\eta^2 \ln \left( \frac{m_\eta^2}{\mu^2} \right) + \frac{7}{2} m_\pi^2 \ln \left( \frac{m_\pi^2}{\mu^2} \right) + 3 m_K^2 \ln \left( \frac{m_K^2}{\mu^2} \right) \right. \\ &\quad - 2 \int [x m_\pi^2 + (1-x) m_K^2 - x(1-x) p^2] \ln \left( \frac{x m_\pi^2 + (1-x) m_K^2 - x(1-x) p^2}{\mu^2} \right) dx \\ &\quad - 2 \int [x m_\eta^2 + (1-x) m_K^2 - x(1-x) p^2] \ln \left( \frac{x m_\eta^2 + (1-x) m_K^2 - x(1-x) p^2}{\mu^2} \right) dx \\ &\quad \left. \left. - 4 \int m_\pi^2 \ln \left( \frac{m_\pi^2}{\mu^2} \right) dx \right] \right\}, \end{aligned} \quad (3)$$

$$\begin{aligned} F_A(p^2) &= \frac{4\sqrt{2} m_K}{F_K} (L_9^r + L_{10}^r) + \frac{m_K}{6 F_K^3 (2\pi)^8} [142.65 (m_K^2 - p^2) - 198.3] \\ &\quad - \frac{m_K}{4\sqrt{2} F_K^3 \pi^2} \left\{ (4L_3^r + 7L_9^r + 7L_{10}^r) m_\pi^2 \ln \left( \frac{m_\pi^2}{m_\rho^2} \right) + 3 (L_9^r + L_{10}^r) m_\eta^2 \ln \left( \frac{m_\eta^2}{m_\rho^2} \right) \right. \\ &\quad \left. + 2 (8L_1^r - 4L_2^r + 4L_3^r + 7L_9^r + 7L_{10}^r) m_K^2 \ln \left( \frac{m_K^2}{m_\rho^2} \right) \right\} \\ &\quad - \frac{4\sqrt{2} m_K}{3 F_K^3} \left\{ 2m_\pi^2 (18y_{18}^r - 2y_{81}^r - 6y_{82}^r + 2y_{83}^r + 3y_{84}^r - y_{85}^r + 6y_{103}^r) \right. \\ &\quad + 2m_K^2 (18y_{17}^r + 36y_{18}^r - 4y_{81}^r - 12y_{82}^r + 4y_{83}^r + 6y_{84}^r + 4y_{85}^r - 3y_{100}^r \\ &\quad \left. + 6y_{102}^r + 12y_{103}^r - 6y_{104}^r + 3y_{109}^r) + \frac{3}{2} (m_K^2 - p^2) (2y_{100}^r - 4y_{109}^r + y_{110}^r) \right\}, \end{aligned} \quad (4)$$

where  $C_i^r$ ,  $L_i^r$  and  $y_i^r$  are the renormalized coupling constants. Note that the first terms in Eqs. (3) and (4) correspond to  $F_V$  and  $F_A$  at  $O(p^4)$  <sup>6</sup>, respectively.

In the framework of the LFQM<sup>10</sup>, we obtain<sup>11</sup>

$$F_A(p^2) = 4m_K \int \frac{dz d^2 k_\perp}{2(2\pi)^3} \Phi(z', k_\perp^2) \frac{1}{1-z'} \left\{ \frac{2}{3} \frac{m_u - A k_\perp^2 \Theta}{m_u^2 + k_\perp^2} + \frac{1}{3} \frac{m_s + B k_\perp^2 \Theta}{m_s^2 + k_\perp^2} \right\},$$

$$F_V(p^2) = 8m_K \int \frac{dz d^2 k_\perp}{2(2\pi)^3} \Phi(z', k_\perp^2) \frac{1}{1-z'} \left\{ \frac{2}{3} \frac{m_u - z'(m_s - m_u) k_\perp^2 \Theta}{m_u^2 + k_\perp^2} - \frac{1}{3} \frac{m_s + (1-z')(m_s - m_u) k_\perp^2 \Theta}{m_s^2 + k_\perp^2} \right\}, \quad (5)$$

where the parameters and variables are defined in Ref.<sup>11</sup>.

The numerical values of  $F_{A,V}(p^2)$  in the ChPT of  $O(p^6)$  are plotted in Fig. 1. In these figures, we have also included the results in the ChPT at  $O(p^4)$ . Explicitly, we find that  $F_{V(A)}(p^2 = 0) = 0.0945$  (0.0425), 0.082 (0.034) and 0.106 (0.036) in the ChPT at  $O(p^4)$ , ChPT at  $O(p^6)$  and LFQM, respectively.

The differential decay rate as a function of  $x$  is given by

$$\frac{d\Gamma}{dx} = \frac{m_K^5}{64\pi^2} \alpha G_F^2 \sin^2 \theta_c A(x) \quad (6)$$

where the function of  $A(x)$  is given in Ref.<sup>11</sup>. By integrating out the variable  $x$  in Eq. (6), in Table 1 we give the decay branching ratio of  $K^+ \rightarrow e^+ \nu_e \gamma$ . Here, as the IB term diverges at the limit of  $x \rightarrow 0$  corresponding to  $p^2 \rightarrow p_{max}^2 = m_K^2$ , we have used the cuts of  $x = 0.01$  and 0.1, respectively. With the cuts, from Table 1 we see that the IB contributions are much smaller than the  $SD^\pm$  ones, which are insensitive to the cut. In Fig. 2, we also display the spectrum of the differential decay branching ratio in the ChPT at both  $O(p^4)$  and  $O(p^6)$  and the LFQM. From Fig. 2, we see that in the region of  $x < 0.7$  or  $E_\gamma < 173$  MeV, the decay branching ratio in the LFQM is much smaller than that in the ChPT at  $O(p^6)$ . On the other hand, in the region of  $x > 0.7$  the statement is reversed. However, if we only consider the contributions in the ChPT at  $O(p^4)$ , the conclusion is weaker. It is clear in the future data analysis such as the one at the experiment BNL-E949<sup>7</sup>, one could concentrate on these two regions to find out which model is preferred.

We have studied the axial-vector and vector form factors of the  $K^+ \rightarrow \gamma$  transition in the LFQM and ChPT of  $O(p^6)$ . Based on these form factors, we have

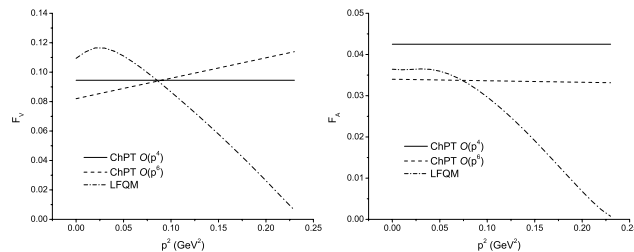


Fig. 1.  $F_{V,A}(p^2)$  as functions of the transfer momentum  $p^2$ .

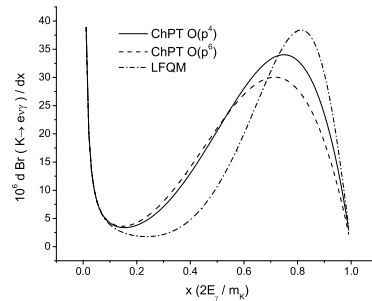
Table 1. The decay branching ratio of  $K^+ \rightarrow e^+ \nu_e \gamma$  (in units of  $10^{-5}$ ).

Model	$x$ Cut	IB	SD <sup>+</sup>	SD <sup>-</sup>	Total
ChPT at $O(p^4)$	0.01	$1.65 \times 10^{-1}$	1.34	$1.93 \times 10^{-1}$	1.70
	0.1	$0.69 \times 10^{-1}$	1.34	$1.93 \times 10^{-1}$	1.60
ChPT at $O(p^6)$	0.01	$1.65 \times 10^{-1}$	1.15	$2.58 \times 10^{-1}$	1.57
	0.1	$0.69 \times 10^{-1}$	1.15	$2.58 \times 10^{-1}$	1.47
LFQM	0.01	$1.65 \times 10^{-1}$	1.12	$2.59 \times 10^{-1}$	1.54
	0.1	$0.69 \times 10^{-1}$	1.12	$2.59 \times 10^{-1}$	1.44

calculated the decay branching ratio of  $K^+ \rightarrow e^+ \nu_e \gamma$ . We have demonstrated that the SD parts give the dominant contributions to the decay in the whole allowed region of the photon energy except the low endpoint. Future precision experimental measurements on the decay spectrum <sup>7</sup> should give us some useful information to determine the SD contributions as well as the vector and axial-vector form factors.

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Fig. 2. The differential decay branching ratio as a function of  $x = 2E_\gamma/m_K$ .